ON RAY FAIR'S MODEL OF EXTRA MARITAL AFFAIRS IN NIGERIA: BAYESIAN PERSPECTIVE

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Abstract: There are so many factors that could result in extra marital affairs in the society. The well-known procedure in the study of extramarital affairs is a classical maximum likelihood method. A major drawback of classical estimation is the lack of incorporation of vital information into phenomena being modelled. This study examines Ray Fair's model of extramarital affairs with Bayesian method of estimation through Gibbs sampler. The Gibbs sampler was used to fit a probit model that involves latent data. The results demonstrate that the Bayesian procedure is able to produce reliable results than other estimators with smaller errors.

Keywords: Extra marital affairs, Bayesian, classical, Gibbs sampler, probit model

Introduction

Sexual infidelity is viewed as taboo in most culture (Nath, 2011). Sexual infidelity also known as extramarital affairs is the sexual or romantic involvement of one's spouse with other person out of marriage. This can be known or unknown to the husband or wife. Earlier work on extra marital affairs by Becker (1973) looked at allocation of a person's time between time spent in household activities and time spent in market activities in relation to extra marital affairs.

Ray Fair in 1978 proposed a model to capture extra marital affairs behaviour of the society. This model explained different allocation of individual's time and other variables. The time measures the activities of household and non-household members and it was observed that leisure time spent by non-household members plays a great role in their lives, and it is unfortunate that this fact has received so little attention by economists. There have been so many follow-up on the Ray fair's model on extramarital affairs, these include; Chernozhukov and Hong (2002), Groot and Brink (2002), Elmslie and Tebaldi(2008), Liu (2008), Nath (2011), Jahan *et al*. (2017), and Mtenga *et al*. (2018) among others.

The causes behind extramarital affairs from an economic point of view was statistically tested using test of hypothesis for number of affairs which depend on demographic characteristics of the population such as gender, age, education, occupation, years married, number of children, satisfaction with married life and degree of religiousness (Nath, 2011). It was identified that more religiousness, higher marital satisfaction and increase in age make people less likely to have extramarital affairs; while increase in years married raises chances of infidelity.

Three-step censored Quantile Regression estimator was proposed by Chernozhukov and Hong (2002) to measure extra marital affairs. A separation restriction was placed on the censoring probability and was found to be useful in samples of small sizes and models with many independent variables. Their approach was applied and compared with that of Fair's model of extra marital example. Elmslie and Tebaldi (2008) also developed an economic model and analyzed the question on how cheating habits differ between women and men. The probit model was used to assess some of the respondents' characteristics. Those characteristics such as age, social class and spouse's educational attainment were identified as factors affecting women's behaviour towards infidelity. It was also find out that women and men respond differently to both the costs and benefits of having an affair. Alternative hypotheses were tested by Groot and Brink (2002) on the relationship between concepts namely; age and education gaps between partners and life. They concluded that for both men and women, satisfaction with marriage and life in general will increase as household income increases, and women are more satisfied when there is an education gap with their spouse.

A stochastic optimization model was used by Liu (2008) to explain why despite the presence of punishments and deterrents, extramarital affairs are still a common occurrence in society. The reasons and effects of extramarital affairs among married adults in Bangladesh were examined by Jahan *et al*. (2017). Their study was based on the information gathered through newspapers, books, and journals published between 1980 and 2016. They concluded that extra marital affairs not only have negative impact on moral and psychological development of the people but also lead to sexually transmitted diseases among people. A cross-sectional sequential explanatory mixed method design was employed using a logistic regression by Mtenga *et al*. 2018 to find the association between extramarital affairs and HIV status in MZIMA project community surveillance representative sample of 3884 married couples aged 15 years and above in Tanzania. It was found out that there is a significant association between extramarital affairs and HIV infection among the women.

These aforementioned studies carried out on extramarital affairs considered the use of classical method of estimation. In this work, we introduce the concept of Bayesian technique for estimation purpose of Ray Fair's model of extramarital affairs. Bayesian technique allows new information to be combined with existing information especially with the use of prior information (Adepoju and Ojo, 2018; Ojo, 2020). It also examines some distinct patterns that exist between demographics and extramarital affairs via probit model. Apart from economic point of view considered by other authors which affects extra marital affairs, there is need to look at the effects and causes of other behavioural view point such as the use of contraceptive and involvement in abortion among other things on extra marital affairs, so that action can be taken to arrest such factor.

Brief overview of Ray Fair's model

A Ray Fair's model is a theoretical model that encompasses on extramarital affairs. It was developed for utility model that allocated a person's time among three activities namely; time spent for works, time spent with spouse and time spent with paramour. The study suggested that further tests of the model should be performed by other researchers especially if other class of data is available. However, the model did not take into account of social class or the race of their sample population, hence there is need to investigate other behavioural viewpoint to know how they influence extramarital affairs especially the use of contraceptive and involvement in abortion.

The Probit model

In Ray Fair's model, a Tobit model was used for estimation. In this model, response variable is observed only if a certain condition(s) is met. As also noted by Maddala (1992), Tobit model is only applicable in a case where latent variable can

take negative values and observed values are a consequence of censoring and non-observability. However, in some applications, normal Cumulative Density Function (CDF) has been found to be useful McFadeen (1973). Probit model however is a kind of model that emanates from normal CDF. It is used when the dependent variable is qualitative in nature.
Consider a regression model given as:

$$y_i^* = x_i'\beta + \varepsilon_i \quad (1)$$

Where: y_i^* is a latent variable; x_i' is an explanatory variable; β is the coefficient; ε_i is the disturbance term
Equation (1) can also be written in matrix form as:

$$y^* = X\beta + \varepsilon \quad (2)$$

Where: $y^* = (y_1^*, \ldots, y_N^*)'$ denotes the latent variable; $\beta = (\beta_1, \ldots, \beta_k)'$ denotes vectors of parameters;

$X = \begin{bmatrix} 1 & x_{12} & \cdots & x_{1k} \\ 1 & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & x_{N2} & \cdots & x_{Nk} \end{bmatrix}$ denotes thematrix of explanatory;

ε has a multivariate normal distribution with mean 0_N and covariance h^{-1}N

The probit model for model in (1) has a relationship between y and y^* written as:

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 1 \\ 0 & \text{if } y_i^* \leq 0 \end{cases} \quad (3)$$

Where: y_i is observable variable; If y^* is known then y will also be known.

Method of estimation

Generally, Ordinary Least Squares (OLS) method is inapplicable to Probit model, because the parameter estimates of the model will be biased and inconsistent, that is, they are biased even asymptotically. However, one can use it if only as a standard or norm of comparison. The popular methods for estimating probit model are Maximum Likelihood (ML) and Berkson's minimum chi-square. We will look at the method of Maximum Likelihood (ML) and introduce the celebrated Bayesian technique in this section for estimation of Ray fair's model of extra marital affairs.

Maximum Likelihood method

This is the most popular method of estimation in probit model. It involves likelihood function to find the point estimators by taking the derivative of the likelihood function with respect to the parameter of interest.
If a given set of data say, y_i and x_i from (1). For the single observation, the conditional probability is given as:

$$\text{P}(y_i = 1|\, x_i) = \Phi(x_i'\beta)$$
$$\text{P}(y_i = 0|\, x_i) = 1 - \Phi(x_i'\beta) \quad (4)$$

We can simply write the likelihood of (4) as:

$$\text{L}(\beta; y_i, x_i) = \prod \phi(x_i\beta)^{y_i} [\,1 - \phi(x_i\beta)]^{1-y_i} \quad (5)$$

Thus, if $y_i = 1$, we have:

$$\text{L}(\beta; y_i, x_i) = \phi(x_i'\beta) \quad (6)$$

And if $y_i = 0$, we have:

$$\text{L}(\beta; y_i, x_i) = 1 - \phi(x_i'\beta) \quad (7)$$

The log likelihood function of equations (6) and (7) is given as:

$$\ln\text{L}(\beta; y_i, x_i) = \sum_{i=1}^{n} [y_i \ln \phi(x_i'\beta) + (1 - y_i)\ ln\ (1 - \phi(x_i'\beta))] \quad (8)$$

It will be observed that the estimator of β that maximizes the function () will be consistent, efficient and also asymptotically normal, hence the log likelihood function will be concave in parameter β. Therefore, we can simply write the asymptotic distribution for estimator $\hat{\beta}$ as:

$$= (n)^{1/2} [(\hat{\beta} - \beta) \rightarrow \text{N}(0, \Psi^{-1})] \quad (9)$$

Where: $\Psi = \text{E}\ [\frac{\varphi^2(x_i'\beta)}{\phi(x_i'\beta)1 - \phi(x_i'\beta)} x_i' x_i]$

Bayesian method

In this subsection, we perform a Bayesian binary choice analysis derivation by using suitable prior distribution to reflect the researcher's belief. Posterior inference in probit model can be done using both the Gibbs sampler and data augmentation (Nasrollahzadeh, 2007; Li and Wang, 2020). In that regard, we are going to derive the augmented posterior and posterior conditional distributions.
The likelihood function of the probit model in (3) can be obtained as follows:
For $y_i = 1$, we have:

$$\text{P}(y_i = 1|\, x_i\beta) = \text{P}(x_i\beta + \varepsilon_i > 0) = \text{P}(\varepsilon_i > -x_i\beta)$$
$$= 1 - \Phi(-x_i\beta)$$
$$= \Phi(x_i\beta) \quad (10)$$

Similarly, when $y_i = 0$,

$$\text{P}(y_i = 0|\, x_i\beta) = 1 - \Phi(x_i\beta) \quad (11)$$

Since we assumed independence across observations, the likelihood function is given as;

$$\text{L}(\beta) = \prod \phi(x_i\beta)^{y_i} [\,1 - \phi(x_i\beta)]^{1-y} \quad (12)$$

The prior distribution of the model in this study is of the form:

$$\beta \sim \text{N}(\mu_\beta, Q_\beta) \quad (13)$$

We derive the augmented posterior distribution in order to simplify the computation of model (Tarmar and Wing, 1987; Albert and Chib, 1993) and it follows as:
Recall rule of probability;

$$\text{P}(\beta, y^*|\, y) = \frac{\text{P}(y,\ y^*|\, \beta)\ \text{P}(\beta)}{\text{P}(y)} \quad (14)$$

Equation (14) can also be written as:

$$\text{P}(\beta, y^*|\, y) \propto \text{P}(y,\ y^*|\, \beta)\ \text{P}(\beta) \quad (15)$$

$\text{P}(y, y^*|\beta)$ is the augmented data density and $\text{P}(\beta)$ represents the prior density.
In order to characterize the distribution, $\text{P}(y,\ y^*|\, \beta)$ in (15) in a more detailed way, we have:

$$\text{P}(y,\ y^*|\, \beta) = \text{P}(y|y^*, \beta)\ \text{P}(y^*|\, \beta) \quad (16)$$

The latent variable y^* can simply be written as:

$$\text{P}(y^*|\, \beta) = \prod_{i=1}^{n} \phi(y_i^*;\ x_i\beta, 1) \quad (17)$$

Hence, the condition for y given parameter β and latent variable y^* when $y_i^* > 0$ and $y_i^* \leq 0$ is given as:

$$\text{P}(y|y^*, \beta) = \prod_{i=1}^{n} [\,I(y_i^* > 0)\, I(y_i = 1) + I(y_i^* \leq 0)\, I(y_i = 0) \quad (18)$$

N.B: $I(.)$ is an indicator function, it takes value of 1 if the statement is true and zero otherwise.
Combining equations (12), (13) and (18), we have:

$$\text{P}(\beta, y^*|y) \propto \text{P}(\beta) \prod_{i=1}^{n} [I(y_i^* > 0)\, I(y_i = 1) + I(y_i^* \leq 0)\, I(y_i = 0)]\, \phi(y_i^*;\ x_i\beta, 1) \quad (19)$$

Hence, the conditional posterior distribution is given as:

$$\beta\ |y^*, y \sim \text{N}(R_\beta r_\beta, R_\beta) \quad (20)$$

Where: $R_\beta = (\,x'x + Q_\beta^{-1})^{-1}$
$r_\beta = x'y^* + Q_\beta^{-1}\mu_\beta$

In order to obtain the conditionals of latent variable, y^* observations, we have:

$$y_i^*|\, \beta, y \propto I(y_i^* > 0)\phi(y_i^*;\ x_i\beta, 1), \text{ if } y_i = 1$$
$$y_i^*|\, \beta, y \propto I(y_i^* \leq 0)\phi(y_i^*;\ x_i\beta, 1), \text{ if } y_i = 0 \quad (21)$$

Thus,

$$y_i^*|\, \beta, y \sim \begin{cases} TN_{(-\infty, 0]}(x_i\beta; 1), & \text{if } y_i = 0 \\ TN_{(0, \infty)}(x_i\beta; 1), & \text{if } y_i = 1 \end{cases} \quad (22)$$

Where: $TN_{[a\ b]}(\mu, \sigma^2)$ represents a normal distribution with mean μ and variance σ^2 truncated at interval [a, b].

The method of estimation using this Bayesian technique involves implementation of Gibbs sampler by iteratively sampling from equations (20) and (22).

Prior specification

In this study, we have 9 X 1 parameter vector of β, and can be specified as:

$$\beta \sim \mathrm{N}(\mu_\beta, Q_\beta)$$

$\beta \sim \mathrm{N}(0, 10^2 I_9)$ and can be written in matrix form as:

$$\mu_\beta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad Q_\beta = 10^2 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (23)$$

The Gibbs sampler will be run for 5000 iterations while the first 1000 will be discarded as burn in replication.

Data description

The data used for this study was collected among Akure women in Nigeria. A multi-stage cluster sampling procedures were employed in selecting the sample population. At the first stage, a probability stratified sampling method was used to divide the sample by Enumeration Areas (EAs) according to 2006 census. Households were selected from each EA; from each household, married women were selected for inclusion. Thus, a sample of 350 women were selected for inclusion in the final sample. The Dichotomous Randomized Response Technique (ADRRT) by Ewemooje *et al*., (2019) was applied for it efficiency in obtaining sensitive information through interviewer administered semi-structured questionnaire on the women to obtain information on extra marital affairs and other sociodemographic variables.

The dependent variable for this study is extra marital affairs measured by "Do you have relationship aside your spouse?" with responses ranging from: "0 = No" and "1 = Yes". The independent variables are sociodemographic factors which were identified as possible associate of extra marital affairs, these are; age, religion, years of schooling, socioeconomic status, relationship status, marital satisfaction, contraceptive use, and involvement in abortion.

Presentation of results and discussion

In this section, results based on ray fair extramarital affairs in Nigeria are presented. Table 1 shows estimates of the variables considered using (the inappropriate estimation method) OLS and (appropriate estimation methods) ML and Bayesian. The OLS estimation method includes 181 individuals who had no affairs and 169 who had one or more affairs. ML and Bayesian methods take into consideration explicitly of every individual. The negative effect in all the three method of estimation especially for education shows that the higher the year of schooling, the less likely the incidence of extramarital affairs. However, Non-religiosity, increased socioeconomic status, not being in the first marriage, contraceptive use and involvement in abortion are more likely to increase extramarital affairs among the women in all the three method of estimation. Using OLS and ML-Probit show reduction in extramarital affairs as the age increases while Bayesian shows increase as age increases with smaller error. Marital dissatisfaction increases extramarital affairs among the women using OLS and ML-Probit while in Bayesian model, marital dissatisfaction reduces extramarital affairs among the women.

Table 1: OLS, probit, and Bayesian estimates of extramarital affairs

Variables	OLS	ML-Probit	Bayesian
Constant	0.2076	-0.826	-1.1999
	(0.1094)	(0.212)	(0.0527)
Age	-0.0005	-0.199	0.0029
	(0.0026)	(0.056)	(2.97E-05)
Religiosity	0.0405	0.059	0.1866
	(0.0581)	(0.113)	(0.0149)
Years of School	-0.0017	-0.012	-0.0040
	(0.0092)	(0.104)	(3.74E-04)
Socioeconomic Status	0.0683	0.113	0.1157
	(0.0495)	(0.098)	(0.0108)
Relationship Status	0.0769	0.139	0.5629
	(0.0573)	(0.114)	(0.0144)
Marital Satisfaction	0.1481	0.212	-0.1896
	(0.0937)	(0.168)	(0.0386)
Contraceptive Use	0.0318	0.042	0.2651
	(0.0537)	(0.107)	(0.0127)
Involvement in Abortion	0.0792	0.122	0.6257
	(0.0661)	(0.126)	(0.0192)

() is the standard error

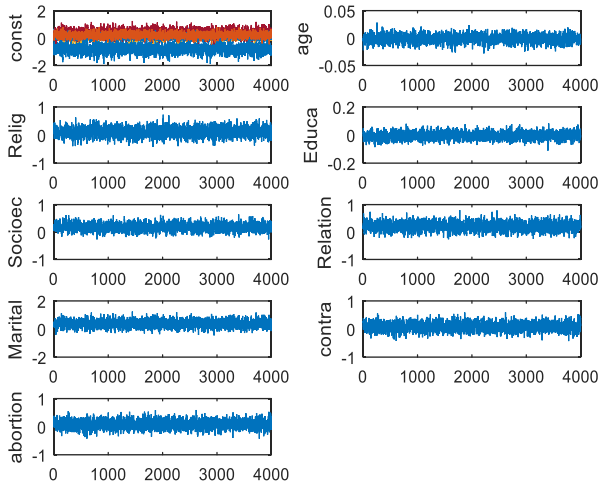

Fig. 1: Plot of variables for the 5000 Gibbs iterations for ray fair model

The draws from posterior simulator are presented in Fig. 1. We obtain nine different plots from nine different chains that were run with different and over-dispersed starting values. It appears as if the simulated draws from all chains appears to settle down and follow the same patter. This proved that the use of 1000 iterations as burn-in-period is more than sufficient for this application.

Conclusion

So many studies on the use of classical method of estimation have been carried out on extramarital affairs model initiated by Ray fair. However, Bayesian method with the use of suitable prior information about the data would further improve the reliability of the results and inferences drawn from such results. In this study, Bayesian method of estimation with the use of Gibbs sampler was considered. The Gibbs sampler was used to fit an extramarital affair model involving latent data through data augmentation approach in order to simplify the computations. This study has underscored the fact that the Bayesian model estimated the extramarital affairs with smaller errors, thus, performs better

than other estimators. Non-religiosity, increased socioeconomic status, not being in the first marriage, contraceptive use and involvement in abortion have been shown to increase extramarital affairs while increased years of schooling reduces extramarital affairs among the women.

Conflict of Interest

Authors declare that there is no conflict of interest related to this study.

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